Precalc Warm Up # 11-2

The 2nd term of a geometric sequence is 3 and the 4th term is 4.32.

Which term will be 8.957952?

HW Questions p. 262

EXERCISES 8.2.3

1. Consider the following sequences:
Arithmetic: 100, 110, 120, 130, . . .
Geometric: 1, 2, 4, 8, 16, . . .
Prove that:
The terms of the geometric sequence will exceed the terms of the arithmetic sequence after the 8th term.
The sum of the terms of the geometric sequence will exceed the sum of the terms of the arithmetic after the 10th term.

2. An arithmetic series has a first term of 2 and a fifth term of 30. A geometric series has a common ratio of –0.5. The sum of the first two terms of the geometric series is the same as the second term of the arithmetic series. What is the first term of the geometric series?

$a_1 = 2 \quad a_5 = 30$

$(1, 2) \quad (5, 30)$

Since arith,

$d = \frac{30-2}{5-1}$

$= \frac{28}{4}$

$= 7$

$a_2 = 2 + 7$

$a_2 = 9$

$r = -0.5$

$g_1 + g_2 = a_2$

$g_1 + g_1(-0.5) = 9$

$g_1(1 - 0.5) = 9$

$g_1 = 18$

3. An arithmetic series has a first term of –4 and a common difference of 1. A geometric series has a first term of 8 and a common ratio of 0.5. After how many terms does the sum of the arithmetic series exceed the sum of the geometric series?

$a_1 = -4$

$d = 1$

$S_n = \frac{n}{2}\left[2a_1 + d(n-1)\right]$

$g_1 = 8$

$r = .5$

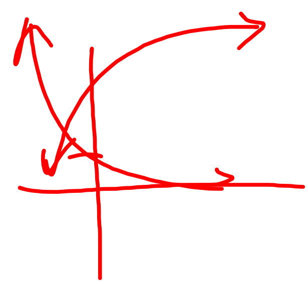
$S_n = \frac{g_1(1-r^n)}{1-r}$

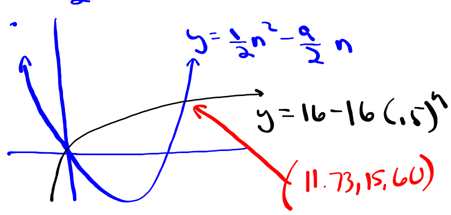
$\frac{n}{2}\left[2(-4) + 1(n-1)\right] > \frac{8(1-.5^n)}{1-.5}$

$\frac{n}{2}(-8+n-1) > \frac{8(1-.5^n)}{.5}$

$\frac{n}{2}(-9+n) > 16(1-.5^n)$

$-\frac{9}{2}n + \frac{1}{2}n^2 > 16 - 16(.5)^n$





By the 12th term

5. Bo-Youn and Ken are to begin a savings program. Bo-Youn saves \$1 in the first week \$2 in the second week, \$4 in the third and so on, in geometric progression. Ken saves \$10 in the first week, \$15 in the second week, \$20 in the third and so on, in arithmetic progression. After how many weeks will Bo-Youn have saved more than Ken?

Bo-Youn
1, 2, 4, ..

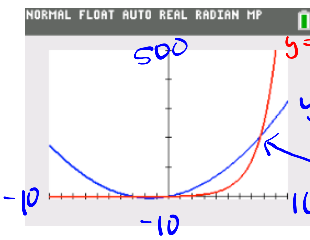
Ken
10, 15, 20, ...

$$S_n = \frac{g_1(1-r^n)}{1-r} \qquad S_n = \frac{n}{2}\left[2a_1 + d(n-1)\right]$$

$$\frac{1(1-2^n)}{1-2} > \frac{n}{2}\left[\underbrace{2(10)+5(n-1)}_{20+5n-5}\right]$$

$$-1+2^n > \frac{n}{2}(15+5n)$$

$$y = \frac{n}{2}(15+5n)$$



7.69, 205.52

in 8 weeks

6. Ari and Chai begin a training program. In the first week Chai will run 10km, in the second he will run 11km and in the third 12km, and so on, in arithmetic progression. Ari will run 5km in the first week and will increase his distance by 20% in each succeeding week.

(a) When does Ari's weekly distance first exceed Chai's?

(b) When does Ari's total distance first exceed Chai's?

Chai
10, 11, 12, ...

Ari
5, 6, 7.2, ...

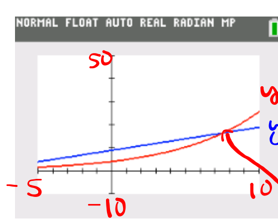
$r = 100\% + 20\% = 120\%$
$= 1.2$

a) $a_n = a_1 + d(n-1) \qquad g_n = g_1 r^{n-1}$

$$10 + 1(n-1) < 5(1.2)^{n-1}$$

$$10 + n - 1 < 5(1.2)^{n-1}$$

$$9 + n < 5(1.2)^{n-1}$$



(7.57, 16.57)

in 8 weeks

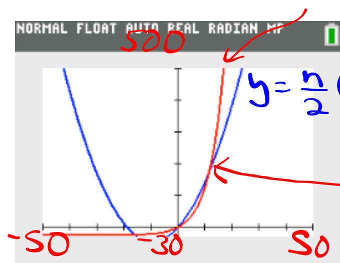
6b.

b) $S_n = \frac{n}{2}\left[2a_1 + d(n-1)\right] \quad < \quad S_n = \frac{g_1(1-r^n)}{1-r}$

$\frac{n}{2}\left[2(10)+1(n-1)\right] \qquad \frac{5(1-1.2^n)}{1-1.2}$

$20+n-1$

$\frac{n}{2}(19+n) < -25(1-1.2^n)$



11.3, 171.19

$\boxed{\text{in 12 weeks}}$

7. The Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, . . . in which each term is the sum of the previous two terms is neither arithmetic nor geometric. However, after the eighth term (21) the sequence becomes approximately geometric. If we assume that the sequence is geometric:

(a) What is the common ratio of the sequence (to four significant figures)?

(b) Assuming that the Fibonacci sequence can be approximated by the geometric sequence after the eighth term, what is the approximate sum of the first 24 terms of the Fibonacci sequence?

This is a fun problem; I'll go over it with you sometime soon!
It won't be on a homework quiz.

$16 + 8 + 4 + 2 + 1 + \ldots\ldots$

Find the sum of the first $r = \frac{1}{2}$

5 terms	6 terms	10 terms	20 terms	100 terms
31	31.5	31.97	≈ 31.99996	≈ 32

$$\frac{16\left(1-\left(\frac{1}{2}\right)^{10}\right)}{\underbrace{1-\frac{1}{2}}_{0.5}} \qquad \frac{16\left(1-\left(\frac{1}{2}\right)^{20}\right)}{1-\frac{1}{2}} \qquad \frac{16\left(1-\left(\frac{1}{2}\right)^{100}\right)}{1-\frac{1}{2}}$$

$16 + 8 + 4 + 2 + 1 + \ldots\ldots$

Add this series to infinity (and beyond!)

$$S_\infty = \frac{16\left(1-\cancel{\left(\frac{1}{2}\right)^{\infty}}^{\,0}\right)}{1-\frac{1}{2}}$$

$$= \frac{16(1-0)}{0.5}$$

$$= 32$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$$

$$\frac{1}{2} = \frac{1}{2^{100}}$$

$$= \frac{1}{1.26 \times 10^{30}} \Rightarrow 0$$

If the series is geometric and the |ratio| < 1,

we say it converges

$$S_\infty = \frac{g_1(1-r^\infty)}{1-r} = \frac{g_1(1-0)}{1-r} = \boxed{\frac{g_1}{1-r}}$$

But this only works if |r| < 1. Why?

Use our new formula:

16 + 8 + 4 + 2 + ... =

Can we add

1 + 2 + 4 + 8 + 16 + ...

to infinity?

$r = 2$

$|r| \not< 1$

$$S_\infty = \frac{1(1-2^\infty)}{1-2}$$

$$= \frac{1(1-\infty)}{-1}$$

Find S_∞

1000 + 100 + 10 + 1 + .1 + ...

$r = \frac{1}{10}$ $\qquad S_\infty = \frac{1000}{1-0.1}$

$= 1{,}111.\overline{1}$

9 - 6 + 4 - 8/3 + ...

$r = -\frac{6}{9} = \frac{-2}{3}$ $\qquad S_\infty = \frac{9}{1+\frac{2}{3}}$

$= 9 \cdot \frac{3}{5}$

$= \frac{27}{5}$

Write the recurring decimal as a mixed number.

2.5454... which the book writes as $2.\dot{5}\dot{4}$ $\quad 2.\overline{54}$

$2 + \frac{54}{100} + \frac{54}{10000} + \frac{54}{} + \ldots$

$0.54 + 0.0054 + 0.000054 + \ldots$

$g_1 = \frac{54}{100}$

$r = \frac{1}{100}$

$2.\overline{54} = 2 + \frac{\frac{54}{100}}{1 - \frac{1}{100}}$

$2 + \frac{54}{\cancel{100}} \cdot \frac{\cancel{100}}{99}$

$2\frac{54}{99}$

Write as a fraction.

0.188888...

$$0.1 + \underbrace{\frac{8}{100} + \frac{8}{1{,}000} + \frac{8}{10{,}000} + \cdots}$$

$$g_1 = \frac{8}{100} \qquad r = \frac{1}{10}$$

$$0.1 + S_\infty$$

$$0.1 + \frac{\frac{8}{100}}{1 - \frac{1}{10}}$$

$$\frac{1}{10} + \frac{\frac{8}{100}}{\frac{9}{10}} \longrightarrow \frac{9}{9}\cdot\frac{1}{10} + \frac{8}{\cancel{100}_{10}}\cdot\frac{\cancel{10}}{9}$$

$$\frac{9}{90} + \frac{8}{90}$$

$$\boxed{\frac{17}{90}}$$

HW: SL book

for #1i) 27, 9, 3, 1, $\frac{1}{3}$, ...

p. 264 #1, 2, 4-7, 10a

(10a is wrong in back)

Group Quiz Wednesday: Partial Fraction Decomposition